On The Structure and Realizability of Some Reconfiguration Graphs

Description. For the last few decades, *Combinatorial Reconfiguration* has emerged in different areas of computer science. In a *reconfiguration variant* of a computational problem (e.g., SATISFIABILITY, INDEPENDENT SET, VERTEX-COLORING, etc.), a *transformation rule* that describes an adjacency relation between *feasible solutions* (e.g., satisfying truth assignments, independent sets, proper vertex-colorings, etc.) of the problem is given. Another way of looking at these reconfiguration problems is via the so-called *reconfiguration graph* (or *solution graph*)—a graph whose nodes are feasible solutions and two nodes are adjacent if one can be obtained from the other by applying the given rule exactly once. A typical example is the well-known classic Rubik's cube puzzle, where each configuration of the cube corresponds to a feasible solution, and two configurations (solutions) are adjacent if one can be obtained from the other by rotating a face of the cube by either 90, 180, or 270 degrees. A classic question is whether there exists a path from an arbitrary node to the one where each face has only one color. In other words, starting from an arbitrary configuration of the Rubik's cube, whether you can rotate its faces so that you finally obtain the configuration where each face has only one color. Readers are referred to the surveys [1, 2, 3] and the wiki page `http://reconf.wikidot.com/` for more details on recent developments in this research area.

Goal. We study the structural properties and realizability of different reconfiguration graphs for some classic computational problems. The main challenge is that the size of such reconfiguration graphs is often really huge (e.g., imagine the number of configurations of a Rubik's cube).

Basically, we focus on the following questions and their extensions:

1. Which *properties* do these reconfiguration graphs satisfy?
2. Which *graph classes* do these reconfiguration graphs belong to?

We remark that these questions have been well-investigated for some computational problems such as DOMINATING SET and VERTEX-COLORING, as surveyed in [3]. However, the questions for several other computational problems remain unsolved and are of great interest from the theoretical viewpoint.

Prerequisites. Basic knowledge of graph theory (which can probably be obtained from an undergraduate-level course in discrete mathematics or graph theory).

References.

[1] Jan van den Heuvel. The Complexity of Change. In: *Surveys in Combinatorics*. Vol. 409. London Mathematical Society Lecture Note Series. Cambridge University Press, 2013, pp. 127–160. doi:10.1017/cbo9781139506748.005

[2] Naomi Nishimura. Introduction to Reconfiguration. In: *Algorithms* 11.4, 52, (2018). doi:10.3390/a11040052

[3] C.M. Mynhardt and S. Nasserasr. Reconfiguration of colourings and dominating sets in graphs. In: *50 years of Combinatorics, Graph Theory, and Computing*. Ed. by Fan Chung, Ron Graham, Frederick Hoffman, Ronald C. Mullin, Leslie Hogben, and Douglas B. West. 1st edition. CRC Press, 2019, pp. 171–191. doi:10.1201/9780429280092-10

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