Linear-Time Algorithm for Sliding Tokens on Trees

Erik D. Demaine[1], Martin L. Demaine[1], Eli Fox-Epstein[2], Duc A. Hoang[3], Takehiro Ito[4], Hirotaka Ono[5], Yota Otachi[3], Ryuhei Uehara[3], and Takeshi Yamada[3]

[1]Massachusetts Institute of Technology, USA. {edemaine, mdemaine}@mit.edu
[2]Brown University, USA. ef@cs.brown.edu
[3]JAIST, Japan. {hoanganhduc, otachi, uehara, tyama}@jaist.ac.jp
[4]Tohoku University, Japan. takehiro@ecei.tohoku.ac.jp
[5]Kyushu University, Japan. hirotaka@econ.kyushu-u.ac.jp

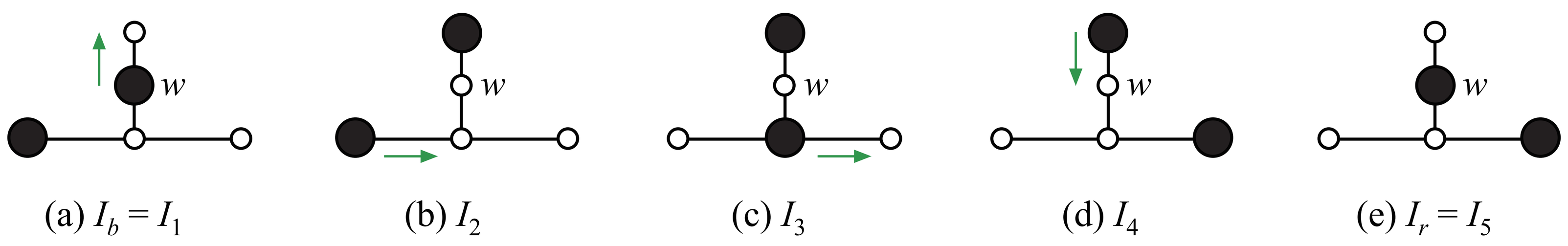
Abstract

Suppose that we are given two independent sets $\mathbf{I}_b$ and $\mathbf{I}_r$ of a graph such that $|\mathbf{I}_b| = |\mathbf{I}_r|$, and imagine that a token is placed on each vertex in $\mathbf{I}_b$. Then, the SLIDING TOKEN *problem is to determine whether there exists a sequence of independent sets which transforms $\mathbf{I}_b$ into $\mathbf{I}_r$ so that each independent set in the sequence results from the previous one by sliding exactly one token along an edge in the graph. This problem is known to be PSPACE-complete even for planar graphs, and also for bounded treewidth graphs.*

In this poster, we show that the problem is solvable for trees in linear time.

1. Examples

1.1 A YES-instance



A YES-instance, where $\mathbf{I}_1 \overset{T}{\leftrightsquigarrow} \mathbf{I}_5$. Token on w makes detour.

1.2 A NO-instance

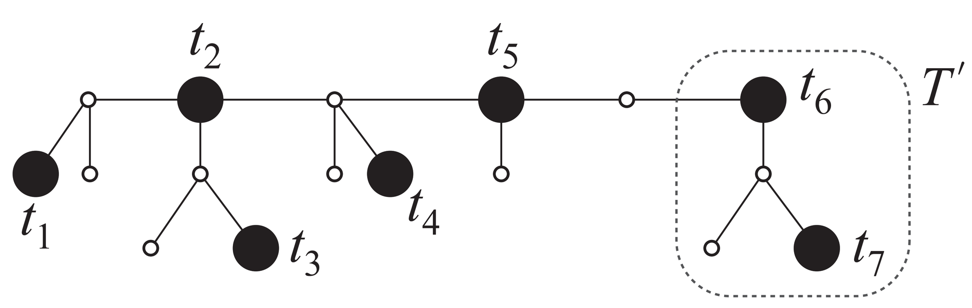
A NO-instance for the SLIDING TOKEN problem.

2. Sliding tokens on trees (ISAAC 2014)

Theorem 1. *The* SLIDING TOKEN *problem can be solved in time $O(n)$ for any tree T with n vertices.*

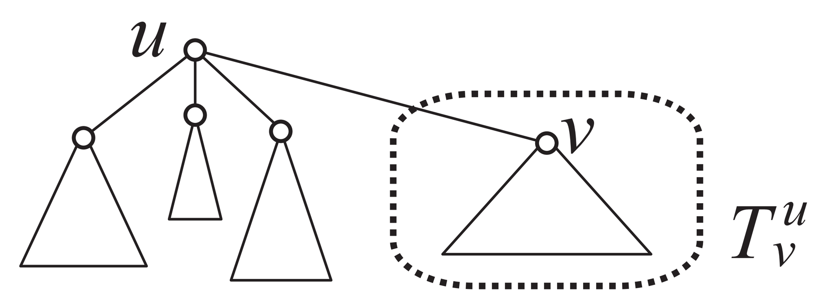
2.1 Rigid tokens

Intuitively, a token on $v \in \mathbf{I}$ is $(T, \mathbf{I})$-rigid if it cannot be slid at all.



An independent set $\mathbf{I}$ of a tree T, where t_1, t_2, t_3, t_4 are $(T, \mathbf{I})$-rigid tokens and t_5, t_6, t_7 are $(T, \mathbf{I})$-movable tokens. For the subtree T', tokens t_6, t_7 are $(T', \mathbf{I} \cap T')$-rigid.

2.2 Determine all rigid tokens

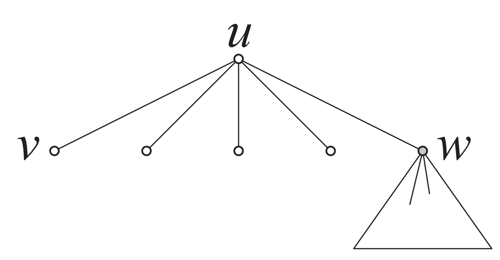


Subtree T_v^u in the whole tree T.

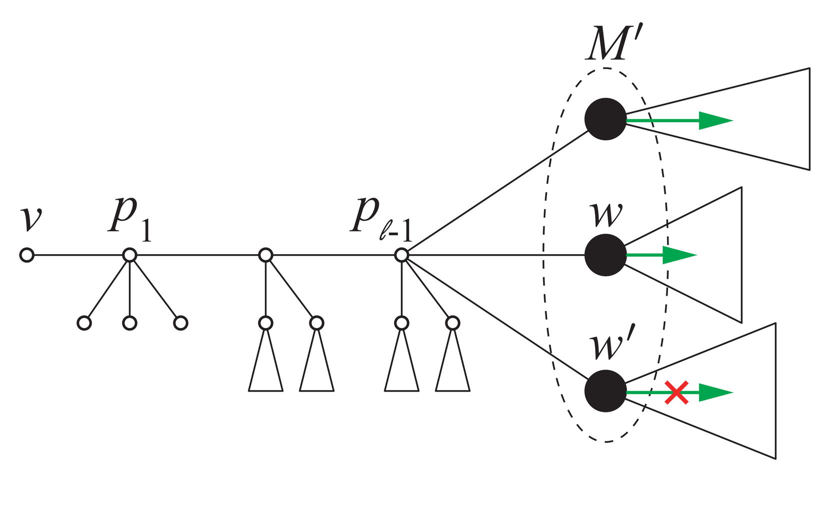
(a) A $(T, \mathbf{I})$-rigid token on u, and (b) a $(T, \mathbf{I})$-movable token on u.

Observation: If the set of movable tokens is not empty, then there is at least one movable token which can be immediately slid to one of its neighbors.

2.3 Instances without rigid tokens



A degree-1 vertex v of a tree T which is *safe*.



Move the nearest token to v (safe degree-1).

3. Discussion

3.1 Extend the concept of "rigid tokens"

A NO-instance for an interval graph. Here all tokens are not rigid, but they are movable in some "restricted area".

3.2 An applicable strategy for solving SLIDING TOKEN problem

1. Characterize the set of tokens which are *movable in some "restricted area"*.
2. Consider the problem's instances when there are no such tokens.

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