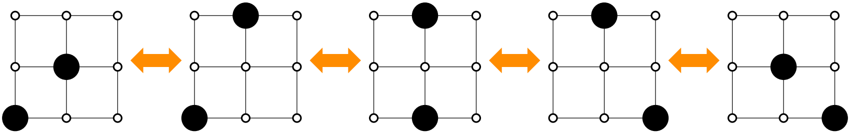
Independent Set Reconfiguration and Related Problems for Some Restricted Graphs



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Outline

1. Background and Motivation
   - Reconfiguration Problems
   - Reconfigurability of Independent Set
2. Our Results
3. Future Works
4. Publications

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Reconfiguration Problems

Reconfiguration Problems $\equiv$ The study of reconfiguration graphs

configurations　　　　reconfiguration graph

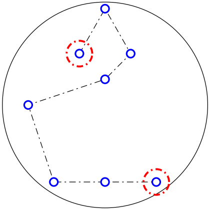
Typical Properties

- adjacency is polynomial testable
- reconfiguration graph is huge

Reconfiguration v.s. Solution Space

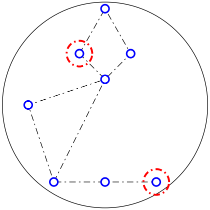
- configurations $\equiv$ feasible solutions of a problem
- reconfiguration rule $\equiv$ small change that preserves the "feasibility" of a solution

Fundamental Questions



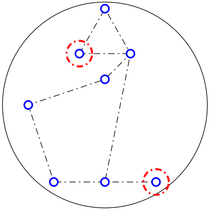
path?

Reachability



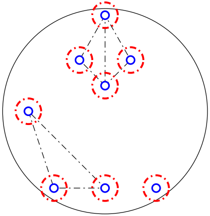
find shortest path?

Shortest Reconf.



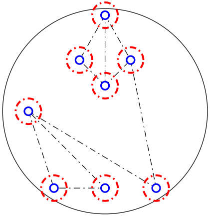
path of length $\leq \ell$?

Bounded Reconf.



connected?

Connectivity



diameter?

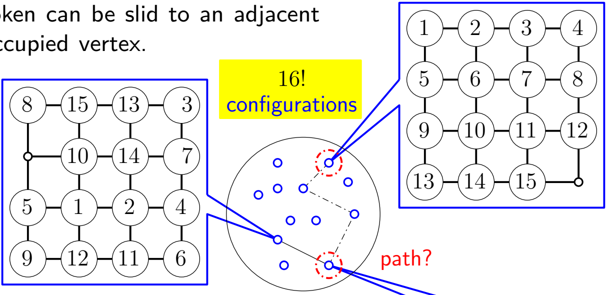
Diameter

Example: 15-PUZZLE

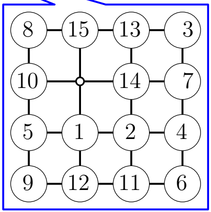
Configuration Placement of 15 tokens labeled $1, 2, \ldots, 15$, on a 4×4 grid.

Adjacency [Token Sliding] A token can be slid to an adjacent unoccupied vertex.

Generalization
[v.d. Heuvel, CoRe'17]
- $(n^2 - 1)$-PUZZLE
- SLIDING-BLOCK
- PEBBLE MOTION
- ROBOT MOTION
- and so on



Problem	Complexity	Reference
REACH.	P	[Johnson and Story 1879]
CONN.	P	[Wilson 1974]
SHORTEST RECONF.	NP-complete	[Ratner and Warmuth 1990]
BOUNDED RECONF.	NP-complete	[Goldreich 2011]
DIAM.	P	[Kornhauser et al. 1984]

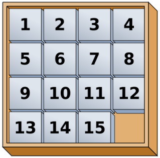


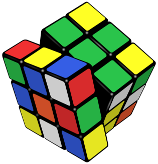
Motivation

Theory Understand the solution space of a problem.

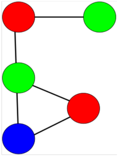
Application Model real-world situations involving movement and change.

- Games & Puzzles.

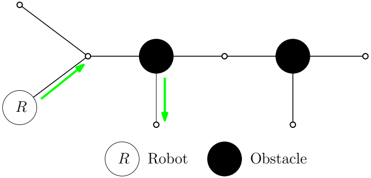




- Frequency Re-Assignment.



- Robot Motion.



Reconfiguration Problems

An Incomplete History [Nishimura, CanaDAM'17]

Before 2011	Lots of work not called reconfiguration
2011	Reconfiguration framework [Ito et al. 2011] ◦ Study REACHABILITY questions ◦ Several classic NP-complete problems have PSPACE-complete reconfiguration variants ◦ Several problems in P whose reconfiguration variants are also in P
Since 2011	Lots of work called reconfiguration (and also lots of work not called reconfiguration)

Surveys on Reconfiguration:

Jan van den Heuvel (2013). "The complexity of change". In: *Surveys in Combinatorics 2013*. Ed. by Simon R. Blackburn et al. London Mathematical Society Lecture Note Series. Cambridge University Press, pp. 127–160. DOI: 10.1017/CBO9781139506748.005

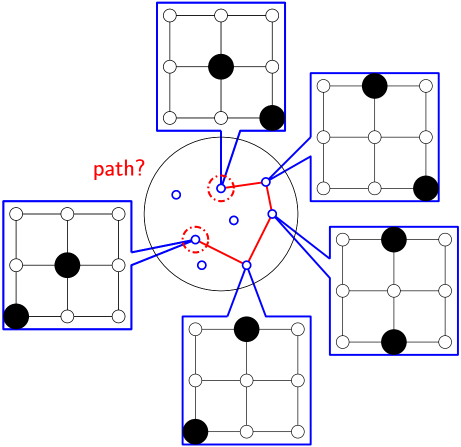
Naomi Nishimura (2018). "Introduction to Reconfiguration". In: *Algorithms* 11.4. DOI: 10.3390/a11040052

Reconfiguration Web Portal: `http://www.ecei.tohoku.ac.jp/alg/core/`

Reconfigurability of Independent Set

SLIDING TOKEN [Hearn and Demaine 2005]

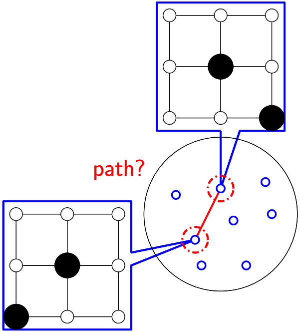
Configuration	Independent set (viewed as a set of tokens) of a graph
Adjacency [Token Sliding]	A token can be slid to an adjacent unoccupied vertex
Problem	REACHABILITY



Reconfigurability of Independent Set

Token Jumping [Kamiński et al. 2012]

Configuration	Independent set (viewed as a set of tokens) of a graph
Adjacency [Token Jumping]	A token can be moved to an unoccupied vertex
Problem	Reachability



Multiple Token Jump [de Berg et al. 2016]

Configuration	Independent set (viewed as a set of tokens) of a graph
Adjacency	p tokens can be moved simultaneously to unoccupied vertices
Problem	Find smallest p such that any two independent sets of equal size are connected by a path in the reconfiguration graph

Reconfigurability of Independent Set

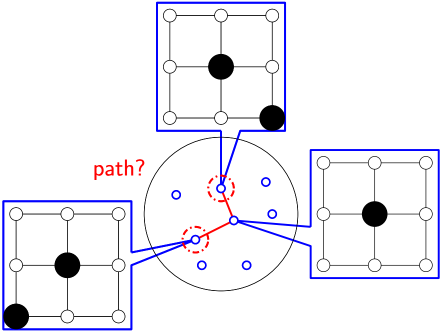
TOKEN ADDITION AND REMOVAL [Ito et al. 2011]

Configuration Independent set (viewed as a set of tokens) of a graph

Adjacency A token can be added or removed

[Token Addition and Removal] (♯ remaining tokens $\geq k$)

Problem REACHABILITY



REACHABILITY problems for Independent Set

Graph	Adjacency TS	Adjacency TAR	Adjacency TJ	Complexity	Reference
planar ∩ maximum degree 3	◯	◯	◯	PSPACE-complete	[Hearn and Demaine 2005]
general line ⇐ MATCHING RECONF.	◯	◯ ◯	◯ ◯	PSPACE-complete P	[Ito et al. 2011]
perfect even-hole-free cograph	◯ ◯	◯ ◯	◯ ◯	PSPACE-complete P P	[Kamiński et al. 2012]
cograph		◯	◯	P	[Bonsma 2014]
bounded bandwidth	◯	◯	◯	PSPACE-complete	[Wrochna 2014]
claw-free (⊃ line)	◯	◯	◯	P	[Bonsma et al. 2014]
tree (⊂ even-hole-free)	◯	◯	◯	P	[Demaine et al. 2014]
bipartite permutation bipartite distance-hereditary	◯ ◯			P	[Fox-Epstein et al. 2015]
planar ∩ maximum degree 3 ∩ bounded bandwidth	◯	◯	◯	PSPACE-complete	[van der Zanden 2015]
cactus		◯	◯	P	[Mouawad et al. 2014]
cactus	◯			P	[Hoang and Uehara 2016]
interval (⊂ even-hole-free)	◯	◯	◯	P	[Bonamy and Bousquet 2017]
bipartite	◯	◯	◯	PSPACE-complete (TS) NP-complete (TAR, TJ)	[Lokshtanov and Mouawad 2018]

* Main Parts of Our Results

For other problems, see [Nishimura 2018, Section 4]

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Hereditary classes

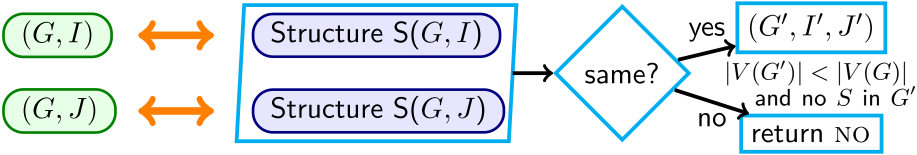
Main Results → Consequences

Graph	REACHABILITY	DIAMETER	**Reference**
trees	$O(n)$	$O(n^2)$	ISAAC 2014 Theor. Comp. Sci. 600, 132–142
cactus graphs	$O(n^2)$	$O(n^2)$	ISAAC 2016

* Here, n is the number of vertices of the corresponding graph.

For an instance (G, I, J) of SLIDING TOKEN, where G is one of the graphs above and I, J are independent sets of G.

1. One can identify all structures that forbid the existence of a path between I and J in the corresponding reconfiguration graph in polynomial time.
2. Without these forbidden structures, such a path exists iff $|I| = |J|$.



What next?

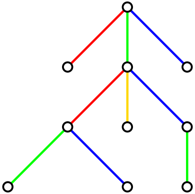
In the rest of this section, we shall present:

1. How to apply the general framework for trees.
2. High-level idea regarding how to identify forbidden structures for cactus graphs.

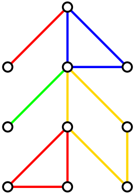
What are trees, cactus graphs?

A block of a graph is either an edge or a maximal 2-connected subgraph. A graph can be decomposed into a collection of blocks, where any two blocks share at most one common vertex. Intuitively, one can view

- a tree as a graph whose block is an edge;
- a cactus graph as a graph whose block is either an edge or a cycle.



(a) A tree



(b) A cactus graph

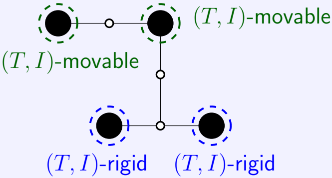
Figure: An example of (a) a tree, (b) and a cactus graph. Two blocks sharing the same vertex are colored by distinct colors.

Sliding Token for trees

For an instance (T, I, J) of Sliding Token, where T is a tree and I, J are independent sets of T.

Forbidden Structure: Rigid Tokens

Intuitively, a token t placed on vertex $u \in I$ is (T, I)-rigid if it cannot be moved at all. If t is not (T, I)-rigid, we say that it is (T, I)-movable.

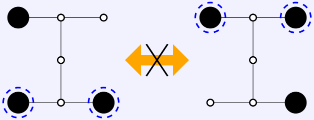


Lemma: *One can find the set $\mathsf{R}(T, I)$ of all (T, I)-rigid tokens in $O(n)$ time, where $n = |V(T)|$.*

Proof Sketch. Among (T, I)-movable tokens, there must be a token that can immediately be moved to one of its neighbors. The removal of such a token does not change the rigidity of other tokens.

Sliding Token for trees

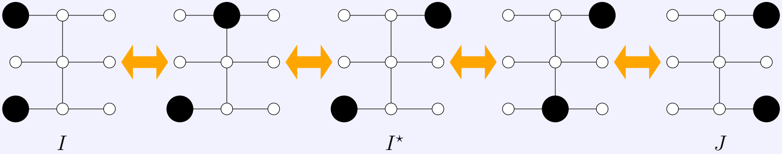
Lemma: *If* $\mathsf{R}(T, I) \neq \mathsf{R}(T, J)$ *then* (T, I, J) *is a* NO*-instance.*



Lemma: *If* $\mathsf{R}(T, I) = \mathsf{R}(T, J) = \emptyset$ *then* (T, I, J) *is a* YES*-instance iff* $|I| = |J|$.

Proof Sketch. Construct an intermediate independent set I^*.

- Pick a safe vertex v, i.e., a vertex of degree-1 whose unique neighbor u has at most one neighbor of degree ≥ 2, and add v to I^*.
- Remove v, u, and the resulting isolated vertices. Repeat the process.



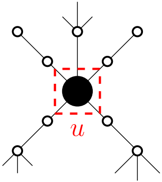
Input: (T, I, J).
Output: YES if there is a path between I, J in the corresponding reconfiguration graph; and NO otherwise.

- **Step 1:** If $\mathsf{R}(T, I) \neq \mathsf{R}(T, J)$, return NO. Otherwise, go to **Step 2**.
- **Step 2:** Let F be the forest obtained by removing all vertices in $N_T[\mathsf{R}(T, I)] = N_T[\mathsf{R}(T, J)]$. If $|I \cap V(F')| = |J \cap V(F')|$ for every component (tree) F' of F then return YES. Otherwise, return NO.

Running Time: $O(n)$, where $n = |V(T)|$.

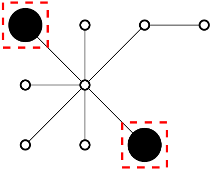
Generalization of rigid tokens

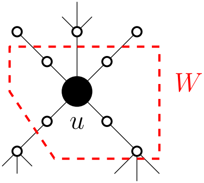
The concept of rigid tokens can be generalized.



(G, I)-rigid token
cannot be moved "out of" $G[\{u\}]$

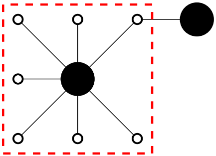
example





(G, I, W)-confined token
cannot be moved "out of" $G[W]$

example



Generalization: trees $\Rightarrow$ cactus graphs

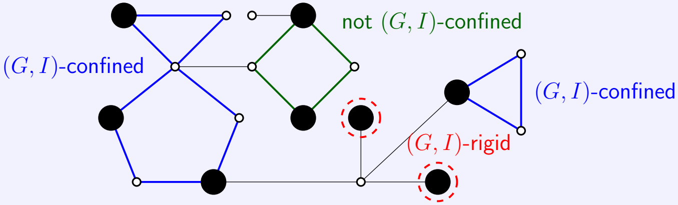
For an instance (G, I, J) of SLIDING TOKEN, where G is a cactus graph and I, J are independent sets of G.

Forbidden Structure

Rigid Tokens — **similar to trees**

Confined Cycles — **new structure**

Intuitively, an induced cycle C of G is (G, I)-confined if $I \cap V(C)$ forms a maximum independent set of C (of size $\lfloor |V(C)|/2 \rfloor$) and all tokens in $I \cap V(C)$ are $(G, I, V(C))$-confined.



Lemma: *One can respectively find the sets* $\mathsf{R}(G, I)$ *and* $\mathsf{C}(G, I)$ *of all* (G, I)*-rigid tokens and* (G, I)*-confined cycles in* $O(n^2)$ *time, where* $n = |V(G)|$.

Lemma: *If either* $\mathsf{R}(G, I) \neq \mathsf{R}(G, J)$ *or* $\mathsf{C}(G, I) \neq \mathsf{C}(G, J)$ *then* (G, I, J) *is a* NO*-instance.*

Lemma: *If* $\mathsf{R}(G, I) = \mathsf{R}(G, J) = \emptyset$ *and* $\mathsf{C}(G, I) = \mathsf{C}(G, J) = \emptyset$ *then* (G, I, J) *is a* YES*-instance iff* $|I| = |J|$.

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Future Works: Other graph classes

It is well-known that

Theorem ([van der Zanden 2015]): *There exists a constant c such that* Independent Set Reconfiguration *(under* TS, TJ, *or* TAR*) is* PSPACE*-complete even for planar graphs of maximum degree 3 and of bandwidth/treewidth/pathwidth/cliquewidth at most c.*

An interesting open question is whether there exists efficient algorithms for solving the problem when the input graph is of small bandwidth/treewidth/pathwidth/cliquewidth. Interesting target graphs are:

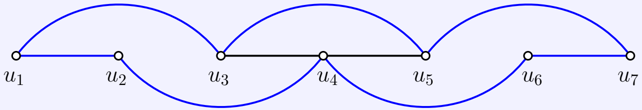
1. Series-parallel graphs ($\equiv$ graphs of treewidth ≤ 2). Cactus graphs is one of its subclasses.
2. Distance-hereditary graphs (whose cliquewidth ≤ 3).
3. Bandwidth-2 graphs ($\equiv$ graphs of bandwidth ≤ 2).

Future Works: Bandwidth-2 graphs

A graph $G = (V, E)$ is a bandwidth-2 (bw2) graph if there exists a one-to-one function $f : V \to \{1, 2, \ldots, |V|\}$ (called a bw2 layout of G) such that its bandwidth $\mathrm{bw}(f) = \max_{uv \in E} |f(u) - f(v)|$ is at most 2. For a layout f, we use the notation $f = (u_1, u_2, \ldots, u_n)$, where $n = |V|$ and $u_i = f^{-1}(u_i)$.

Conjecture: Sliding Token *for bandwidth-2 graphs is in* P.

Lemma ([Makedon et al. 1993]): *Let $G = (V, E)$, $|V| = n \geq 3$, be a* bw2 *biconnected graph. Let $f = (u_1, u_2, \ldots, u_n)$ be a* bw2 *layout of G. Then, the pairs (u_1, u_2), (u_i, u_{i+2}), $1 \leq i \leq n-2$, and (u_{n-1}, u_n) are all edges of G.*



- cycles $\Rightarrow$ bw2 biconnected graphs
- cactus graphs $\Rightarrow$ bw2 graphs

In this thesis, we've claimed that

Theorem: INDEPENDENT SET RECONFIGURATION *for trees (under* TS, TJ, TAR*) is in* P*.*

Moreover,

Theorem ([Kamiński et al. 2012]): SHORTEST RECONFIGURATION *for* INDEPENDENT SET RECONFIGURATION *under* TJ *and* TAR *rules are in* P *for even-hole-free graphs (⊃ trees).*

Interestingly, the following question remains open for trees.

SHORTEST SLIDING TOKEN [Yamada and Uehara 2016]

Configuration	Independent set (viewed as a set of tokens) of a graph
Adjacency [Token Sliding]	A token can be slid to an adjacent unoccupied vertex
Problem	SHORTEST RECONFIGURATION

Theorem ([Yamada and Uehara 2016]): Shortest Sliding Token *for caterpillars* ($\subset$ *trees) is in* P.

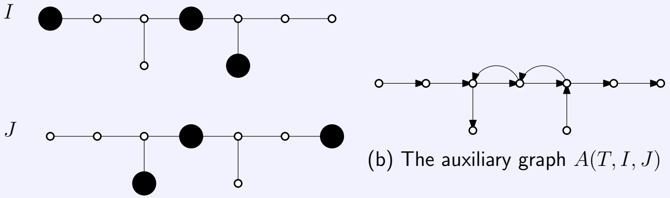
Theorem (in discussion with Amanj Khorramian and Ryuhei Uehara [unpublished]): Shortest Sliding Token *for spiders (i.e., trees having exactly one vertex of degree* ≥ 3*) is in* P.

Future Works: Shortest Reconf. for Trees

Conjecture: Shortest Sliding Token *for trees is in* P.

Key Structure: Auxiliary Graph $A(T, I, J)$

For an instance (T, I, J), we define the corresponding (directed) auxiliary graph $A(T, I, J)$ such that $V(A(T, I, J)) = V(T)$ and $E(A(T, I, J)) = \left\{(x, y) : xy \in E(T) \text{ and } \left|I \cap V(T^x_y)\right| \leq \left|J \cap V(T^x_y)\right|\right\}$, where T^x_y is the subtree of T induced by y and its descendants when regarding x as the root of T.



(a) An instance (T, I, J)

(b) The auxiliary graph $A(T, I, J)$

In general, one may study the structure of the corresponding reconfiguration graph of INDEPENDENT SET RECONFIGURATION (under TS, TJ, TAR). Some interesting problems are:

1. Which graph can be a reconfiguration graph of INDEPENDENT SET RECONFIGURATION?
   - Under TJ/TAR: First studied in [Fatehi et al. 2017].
   - Under TS: Open.
2. Whether the reconfiguration graph and its corresponding original graph belong to the same graph classes?
3. and so on.

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Publications

- Journal
  - Erik D. Demaine, Martin L. Demaine, Eli Fox-Epstein, Duc A. Hoang, Takehiro Ito, Hirotaka Ono, Yota Otachi, Ryuhei Uehara, and Takeshi Yamada: "Linear-Time Algorithm for Sliding Tokens on Trees," *Theoretical Computer Science*, Vol. 600, pp. 132–142 (Jul. 2015).
- International Conference
  - Erik D. Demaine, Martin L. Demaine, Eli Fox-Epstein, Duc A. Hoang, Takehiro Ito, Hirotaka Ono, Yota Otachi, Ryuhei Uehara, and Takeshi Yamada: "Polynomial-Time Algorithm for Sliding Tokens on Trees," Proceedings of the 25th International Symposium on Algorithms and Computation (ISAAC 2014), LNCS 8889, pp. 389–400 (Dec. 2014).
  - Duc A. Hoang, and Ryuhei Uehara: "Sliding Tokens on a Cactus," Proceedings of the 27th International Symposium on Algorithms and Computation (ISAAC 2016), LIPIcs 64, pp. 37:1–37:26 (Dec. 2016).
- Others (not included in this thesis)
  - Eli Fox-Epstein, Duc A. Hoang, Yota Otachi, and Ryuhei Uehara: "Sliding Token on Bipartite Permutation Graphs," Proceedings of the 26th International Symposium on Algorithms and Computation (ISAAC 2015), LNCS 9472, pp. 237–247 (Dec. 2015).
  - Duc A. Hoang, Eli Fox-Epstein, and Ryuhei Uehara: "Sliding Tokens on Block Graphs," Proceedings of the 11th International Conference and Workshops on Algorithms and Computation (WALCOM 2017), LNCS 10167, pp. 460–471 (Mar. 2017).

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