Vietnam Polymath REU Workshop: Abstracts

Packing sets under the special linear groups

Le Quang-Hung

Abstract

Let $SL_2(\mathbb{F}_q)$ be the special linear group with entries in a finite field $\mathbb{F}_q$, $S \subset SL_2(\mathbb{F}_q)$, $E \subset \mathbb{F}_q^2$, and we define $S(E) := \bigcup_{\theta \in S} \theta(E)$. In this talk, we are interested in finding conditions on S and E such that either $|S(E)| \gg q^2$ or $|S(E)| \gg |E|^{1+\epsilon}$ for some $\epsilon > 0$. To study this problem, we introduce and develop a new incidence structure between the group $SL_2(\mathbb{F}_q)$ and pairs of points in $\mathbb{F}_q^2 \times \mathbb{F}_q^2$. As a consequence, we obtain sharp results when the sets are of large size and the first non-trivial results when the sets are of small size.

Surface subgroups of Baumslag doubles along short words

Le Xuan Hoang Tran Nguyen Nam Hung

Abstract

Mikhael Gromov asked the following question in Geometric group theory, Volume 2 (1991): "Does every one-ended word-hyperbolic group have a hyperbolic surface subgroup?". An interesting case is when the group is a Baumslag double of a finite-rank group, which can be reformulated into a graph-theoretic problem via Whitehead graph whose vertices are letters of the alphabet associated with the free group. Kim and Oum (2010), who worked on two different research areas, solved this problem in two cases:

1. The Whitehead graph has exactly four vertices,
2. Each vertex of the Whitehead graph has the same valence.

We develop their arguments and use our computational approaches to discover this problem further, which successfully tackles with two more cases:

1. The number of edges does not exceed the number of vertices plus four,
2. The Whitehead graph has six edges and is simple.

The solvability of systems revisited

Thang Nguyen Duc

Abstract

In this paper, we introduce a new and direct approach to study the solvability of systems of equations generated by bilinear forms. More precisely, let $B(\cdot,\cdot)$ be a non-degenerate bilinear form and E be a set in $\mathbb{F}_q^2$. We prove that if $|E| \gg q^{5/3}$ then the number of triples $(B(x,y), B(y,z), B(z,x))$ with $x, y, z \in E$ is at least cq^3 for some positive constant c. This significantly improves a result due to the fifth listed author (2009).

Random walk on Cayley graphs and unimodular random graphs

Truong Le Gia Khanh Le Thao Mi

Abstract

In this project, we study the upper bounds of the return probability of simple random walks on Cayley graphs of group and unimodular random graphs, as in [Mikolaj Fraczyk, Ben Hayes, Madhu Sudan, and Yufei Zhao, 2024] preprint. Specifically, we will show the transition probabilities $\mathbb{P}(X_n = e)$, or $\mathbb{P}(X_n = o)$ if we consider random rooted graph, satisfy the estimate

$$\mathbb{P}(X_n = e) \leq \rho^n C n^{-\alpha},$$

where ρ is the spectral radius associated with the random walk and $\alpha, C > 0$ are some constants. The striking result to prove that is the estimate on singularity of the spectral measure at the top of infinite unimodular random graphs, which are more general than Cayley graphs of groups. Moreover, we will discuss some facts about lower bounds of return probabilities on free groups, as mentioned in Random walks on infinite graphs and groups (2000).

A Note On Reconfiguration Graphs of Cliques

Phan Huu An Lam Nhat Quan

Abstract

In this project, we explore the characteristics of the reconfiguration graph of cliques, building on the findings of Ito, Ono, and Otachi (2014). The first part of the study examines the relationship between the graph TJ_k and TS_{k-1} presenting an algorithm to derive $TS_{k-1}(G)$ from $TJ_k(G)$ where $k = \omega(G)$. The second part investigates various properties of the $TS_k(G)$ graph including its chromatic number in relation to the Johnson graph, its planarity when G is planar, and the presence of common dense subgraphs with G. Also, some properties of the Simplex Graph, which is known as the origin for the graph $TAR(G)$, will be examined.

Efficient Transformer via Context-Aware Sampling and Convex Characteristics

Le Nhat Minh Nguyen Viet Anh Nguyen Toan Khoa

Abstract

In this paper, we address the limitations of transformers [1] as sequence processing models, specifically the quadratic complexity of self-attention modules [2] and the challenges in training transformers [3]. We introduce two novel mechanisms to mitigate these issues: a neural-guided down-sampling approach for self-attention and a new attention non-linearity that is both linearly scaled and convex. These incremental changes accelerate self-attention computations and simplify hyper-parameter tuning. Additionally, our relative positional encoding method is applicable to various data structures and constraints, such as rotational invariance in 3D point clouds. This makes our model capable of handling diverse data structures including point clouds, graphs, and long-range sequences. As a foundation model, it delivers competitive results across multiple data structures in standard benchmarks, outperforming specialized models in terms of speed and efficiency during inference.

[1]: Vaswani, Ashish; Shazeer, Noam; Parmar, Niki; Uszkoreit, Jakob; Jones, Llion; Gomez, Aidan N; Kaiser, Lukasz; Polosukhin, Illia (2017). "Attention is All you Need". Advances in Neural Information Processing Systems. 30. Curran Associates, Inc.

[2]: Yi Tay, Mostafa Dehghani, Dara Bahri, and Donald Metzler. 2022. Efficient Transformers: A Survey. ACM Comput. Surv. 55, 6, Article 109 (June 2023), 28 pages. https://doi.org/10.1145/3530811

[3]: Liyuan Liu, Xiaodong Liu, Jianfeng Gao, Weizhu Chen, and Jiawei Han. 2020. Understanding the Difficulty of Training Transformers. In Proceedings of the 2020 Conference on Empirical Methods in Natural Language Processing (EMNLP), pages 5747–5763, Online. Association for Computational Linguistics.

LOT Groups and the Whitehead Conjecture

Tran Duc Trong Van Nguyen Dinh Dang Khoa Vo Ke Hoang

Abstract

In this presentation, we will introduce a class of groups called LOT groups, or labelled oriented tree groups. The purpose of studying these groups is to find a counterexample to the Whitehead Conjecture. Some questions we've been considering lately is necessary and sufficient conditions for a LOT group to be abelian, and whether they are torsion-free. We will also introduce our current approach, which is using the Alexander invariant to distinguish between abelian and non-abelian LOT groups.

Counting orthogeodesics on thrice-punctured sphere

Dao Trong Toan

Abstract

In this presentation, we focus on the problem of counting orthogeodesics and prime geodesics on the thrice-punctured spheres. Additionally, we propose a potential algorithm to calculate the Hausdroff dimension of these geodesics in some special cases. We also discuss some observations and related conjectures.

On Hausdorff dimension of projections and distance sets

Le Tran Duy Anh

Abstract

Let $A \subset \mathbb{R}^n$ be an arbitrary set and $e \in S^{n-1}$ be a direction. We define the projection $P_e : \mathbb{R}^n \to \mathbb{R}$ given by $P_e(x) = e \cdot x$ and the distance set $D(A) = \{|x - y| : x, y \in A\}$. If A is a finite set, then the problem with finding the cardinality $|D(A)|$ is known as the Erdos distance problem. In this presentation, we will look at Marstand's result on the dimension of the projection set and Falconer's conjecture and its related theorems on the distance set. We will also consider similar results with the application of Grassmannian submanifold $G(n, m)$.