On The Complexity of Distance-d Independent Set Reconfiguration

Duc A. Hoang[a,b]

[a]VNU University of Science, Hanoi, Vietnam
hoanganhduc@hus.edu.vn

[b]Graduate School of Informatics, Kyoto University, Kyoto, Japan[1]
hoang.duc.8r@kyoto-u.ac.jp

The 17th International Conference and Workshops on Algorithms and Computation
(WALCOM 2023, Hsinchu, Taiwan)

March 22–24, 2023

[1]Until January 31, 2023

Outline

1 Distance-d Independent Set Reconfiguration (DdISR)

2 Observations
- Graph Powers
- Graphs With Bounded Diameter Components

3 General Graphs

4 Chordal Graphs and Split Graphs

5 Open Question: Trees

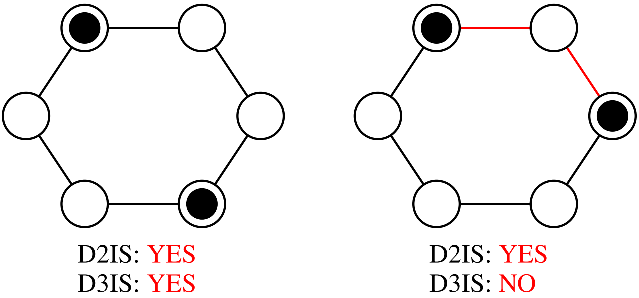
Distance-d Independent Set Reconfiguration (DdISR)

Distance-d Independent Sets (DdISs)

Let G be a simple, undirected graph and let $d \geq 2$ be a fixed integer.

- An *independent set (IS)* of G is a vertex subset where *no two members are joined by an edge*.
- A *distance-d independent set (DdIS)* of G is a vertex subset where *no two members are joined by a path on at most d vertices*.
- Any IS is a D2IS and vice versa.
- Any DdIS is an IS, but an IS may not be a DdIS for $d \geq 3$.

Figure: Some size-2 DdISs of C_6 for $d \in \{2, 3\}$.



Distance-d Independent Set (DdIS)

MAXIMUM DISTANCE-d INDEPENDENT SET (MAXDdIS) ($d \geq 2$)
Input: (G, k)
Question: Is there a DdIS of G having at least k members?

Distance-d Independent Set (DdIS)

MAXIMUM DISTANCE-d INDEPENDENT SET (MAXDdIS) ($d \geq 2$)
Input: (G, k)
Question: Is there a DdIS of G having at least k members?

Table: Computational complexity of MAXDdIS on some graphs.

Graph	d = 2	d ≥ 3
general	NP-C [Garey and Johnson 1979]	NP-C [Kong and Zhao 1993]
bipartite	P (König-Egerváry's Theorem)	NP-C [Eto et al. 2014]
chordal	P [Gavril 1972]	NP-C for odd d P for even d [Eto et al. 2014]

Distance-d Independent Set (DdIS)

MAXIMUM DISTANCE-d INDEPENDENT SET (MAXDdIS) ($d \geq 2$)
Input: (G, k)
Question: Is there a DdIS of G having at least k members?

Table: Computational complexity of MAXDdIS on some graphs.

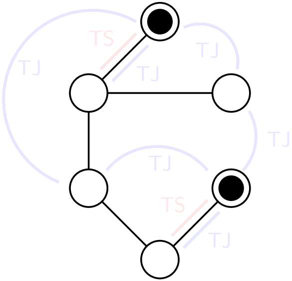
Graph	**d = 2**	**d ≥ 3**
general	NP-C [Garey and Johnson 1979]	NP-C [Kong and Zhao 1993]
bipartite	P (König-Egerváry's Theorem)	NP-C [Eto et al. 2014]
chordal	P [Gavril 1972]	NP-C for odd d P for even d [Eto et al. 2014]

- For other graph classes, see [Katsikarelis et al. 2020]; [Yamanaka et al. 2019]; [Montealegre and Todinca 2016]; [Jena et al. 2018].

Distance-d Independent Set Reconfiguration (DdISR)

Imagine that a token is placed on each vertex of a DdIS A. (Assuming no vertex has more than one token.)

Figure: TS/TJ-moves to obtain a new adjacent D2IS (≡ independent set).

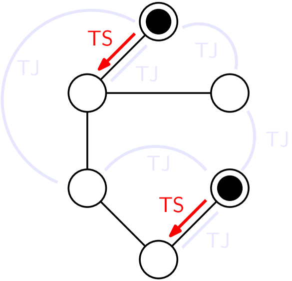


Distance-d Independent Set Reconfiguration (DdISR)

Imagine that a token is placed on each vertex of a DdIS A. (Assuming no vertex has more than one token.)

- A DdIS B is *adjacent to A under Token Sliding* (TS) if it is obtained from A by moving a single token from one vertex to *an unoccupied adjacent vertex*.

Figure: TS/TJ-moves to obtain a new adjacent D2IS (≡ independent set).

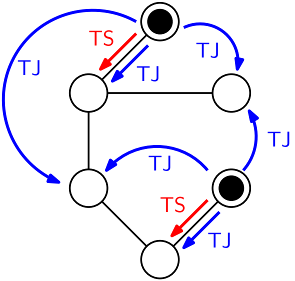


Distance-d Independent Set Reconfiguration (DdISR)

Imagine that a token is placed on each vertex of a DdIS A. (Assuming no vertex has more than one token.)

- A DdIS B is *adjacent to A under Token Sliding* (TS) if it is obtained from A by moving a single token from one vertex to *an unoccupied adjacent vertex*.
- A DdIS B is *adjacent to A under Token Jumping* (TJ) if it is obtained from A by moving a single token from one vertex to *any unoccupied vertex*.

Figure: TS/TJ-moves to obtain a new adjacent D2IS (≡ independent set).

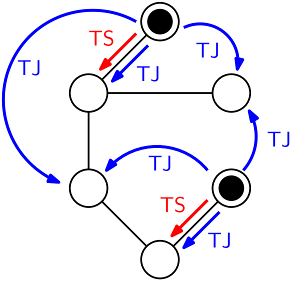


Distance-d Independent Set Reconfiguration (DdISR)

Imagine that a token is placed on each vertex of a DdIS A. (Assuming no vertex has more than one token.)

- A DdIS B is *adjacent to A under Token Sliding* (TS) if it is obtained from A by moving a single token from one vertex to *an unoccupied adjacent vertex*.
- A DdIS B is *adjacent to A under Token Jumping* (TJ) if it is obtained from A by moving a single token from one vertex to *any unoccupied vertex*.

Figure: TS/TJ-moves to obtain a new adjacent D2IS (≡ independent set).



Distance-d Independent Set Reconfiguration (DdISR) ($d \geq 2$) under $\mathsf{R} \in \{\mathsf{TS}, \mathsf{TJ}\}$
Input: (G, I, J, R, d)
Question: Is there a sequence of adjacent DdISs under R between I and J?

Distance-d Independent Set Reconfiguration (DdISR)

Graph	**d = 2**		**d ≥ 3**	
	TS	TJ	TS	TJ
planar	PSPACE-C [Hearn and Demaine 2005]			
general	PSPACE-C [Ito et al. 2011]			
perfect	PSPACE-C [Kamiński et al. 2012]			
even-hole-free	PSPACE-C (⊇ split)	P [Kamiński et al. 2012]		
chordal	PSPACE-C (⊇ split)	P (⊆ even-hole-free)		
split	PSPACE-C [Belmonte et al. 2021]	P (⊆ even-hole-free)		
cograph	P [Kamiński et al. 2012]	P [Bonsma 2016]		
claw-free	P [Bonsma et al. 2014]			
tree	P [Demaine et al. 2015]	P (⊆ even-hole-free)		
bipartite permutation	P [Fox-Epstein et al. 2015]	unknown		
cactus	P [Hoang and Uehara 2016]	P [Mouawad et al. 2018]		
interval	P [Bonamy and Bousquet 2017]	P (⊆ even-hole-free)		
bipartite	PSPACE-C [Lokshtanov and Mouawad 2019]	NP-C [Lokshtanov and Mouawad 2019]		

What happen here?

Distance-d Independent Set Reconfiguration (DdISR)

Graph	$d = 2$		$d \geq 3$	
	TS	TJ	TS	TJ
planar	PSPACE-C [Hearn and Demaine 2005]		PSPACE-C	
general	PSPACE-C [Ito et al. 2011]		PSPACE-C	
perfect	PSPACE-C [Kamiński et al. 2012]		PSPACE-C	
even-hole-free	PSPACE-C (⊇ split)	P [Kamiński et al. 2012]	unknown	PSPACE-C if d is odd unknown if d is even
chordal	PSPACE-C (⊇ split)	P (⊆ even-hole-free)	unknown	PSPACE-C if d is odd P if d is even
split	PSPACE-C [Belmonte et al. 2021]	P (⊆ even-hole-free)	P	PSPACE-C if $d = 3$ P if $d \geq 4$
cograph	P [Kamiński et al. 2012]	P [Bonsma 2016]	P	
claw-free	P [Bonsma et al. 2014]		unknown	
tree	P [Demaine et al. 2015]	P (⊆ even-hole-free)	unknown	P
bipartite permutation	P [Fox-Epstein et al. 2015]	unknown	unknown	
cactus	P [Hoang and Uehara 2016]	P [Mouawad et al. 2018]	unknown	
interval	P [Bonamy and Bousquet 2017]	P (⊆ even-hole-free)	unknown	P
bipartite	PSPACE-C [Lokshtanov and Mouawad 2019]	NP-C [Lokshtanov and Mouawad 2019]	unknown	

Distance-d Independent Set Reconfiguration (DdISR)

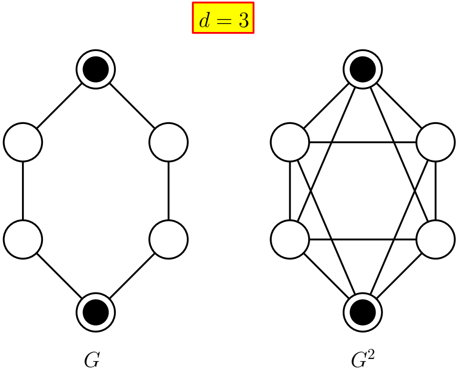
Graph	$d = 2$		$d \geq 3$	
	TS	TJ	TS	TJ
planar	PSPACE-C [Hearn and Demaine 2005]		PSPACE-C	
general	PSPACE-C [Ito et al. 2011]		PSPACE-C	
perfect	PSPACE-C [Kamiński et al. 2012]		PSPACE-C	
even-hole-free	PSPACE-C (⊇ split)	P [Kamiński et al. 2012]	unknown	PSPACE-C if d is odd unknown if d is even
chordal	PSPACE-C (⊇ split)	P (⊆ even-hole-free)	unknown	PSPACE-C if d is odd P if d is even
split	PSPACE-C [Belmonte et al. 2021]	P (⊆ even-hole-free)	P	PSPACE-C if $d = 3$ P if $d \geq 4$
cograph	P [Kamiński et al. 2012]	P [Bonsma 2016]	P	
claw-free	P [Bonsma et al. 2014]		unknown	
tree	P [Demaine et al. 2015]	P (⊆ even-hole-free)	unknown	P
bipartite permutation	P [Fox-Epstein et al. 2015]	unknown	unknown	
cactus	P [Hoang and Uehara 2016]	P [Mouawad et al. 2018]	unknown	
interval	P [Bonamy and Bousquet 2017]	P (⊆ even-hole-free)	unknown	P
bipartite	PSPACE-C [Lokshtanov and Mouawad 2019]	NP-C	unknown	

Observations

Graph Powers

- The *s-th power* of a graph G is the graph G^s with $V(G^s) = V(G)$ and $E(G^s) = \{uv : u, v \in V(G^s) = V(G) \text{ and } dist_G(u, v) \leq s\}$.
- I is a DdIS of G $\Leftrightarrow$ I is an independent set ($\equiv$ D2IS) of G^{d-1}.

Figure: A graph G and its 2-nd power G^2.



Observation

MaxDdIS on G is YES $\Leftrightarrow$ MaxD2IS on G^{d-1} is YES.

Graph Powers

Question

DdISR under R on G is YES $\overset{?}{\Leftrightarrow}$ D2ISR under R on G^{d-1} is YES

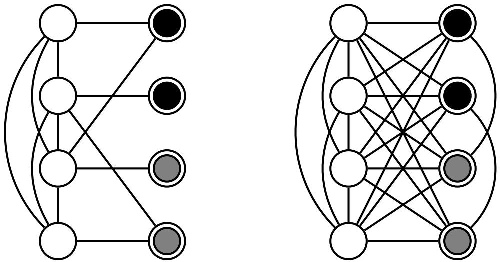
Graph Powers

Question

DdISR under R on G is YES $\overset{?}{\Leftrightarrow}$ D2ISR under R on G^{d-1} is YES

Answer

- R = TJ: *TRUE* (edges in G^{d-1} are "irrelevant")
- R = TS: *FALSE* (edges in G^{d-1} are "relevant")

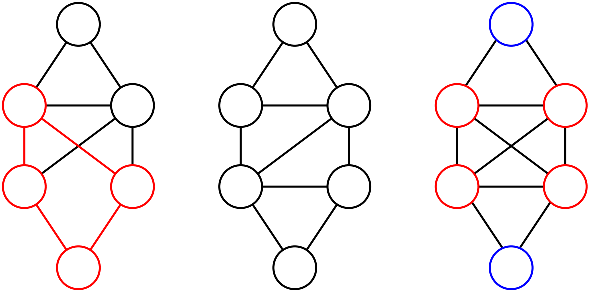


$(G, I, J, \mathsf{TS}, 3)$: NO $\quad$ $(G^2, I, J, \mathsf{TS}, 2)$: YES

Remind: Chordal Graphs and Split Graphs

- G is a *chordal graph* if every cycle C on four or more vertices in G has a chord—an edge joining two non-adjacent vertices in C.
- G is a *split graph* if $V(G)$ can be partitioned into two sets K and S which respectively induce a clique and an independent set.
- Any split graph is also chordal.

Figure: Examples of graphs that are (not) chordal/split.



chordal: NO
split: NO

chordal: YES
split: NO

chordal: YES
split: YES

Graphs With Bounded Diameter Components

Proposition 1

Suppose that any component of G has diameter at most c. DdISR under $\mathsf{R} \in \{\mathsf{TS}, \mathsf{TJ}\}$ *on G is in* P *for any $d \geq c + 1$.*

- Any DdIS of G where $d \geq c + 1$ is of size exactly 1.

Corollary 2

DdISR under $\mathsf{R} \in \{\mathsf{TS}, \mathsf{TJ}\}$ *on split graphs (whose components having diameter ≤ 3) is in* P *for any $d \geq 4$.*

General Graphs

General Graphs

Proposition 3

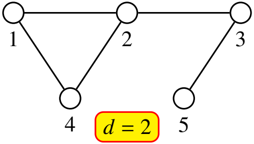
DdISR under TJ *on general graph is* PSPACE*-complete for any* $d \geq 3$.

- *Reduction from D2ISR under* TJ *on general graphs*—a PSPACE-complete whose complexity was shown in [Ito et al. 2011].

General Graphs

- Reduction from D2ISR under TJ on general graphs.

Figure: Reduction $(G, I, J, \mathsf{TJ}, 2) \Rightarrow (G', I, J, \mathsf{TJ}, d)$.



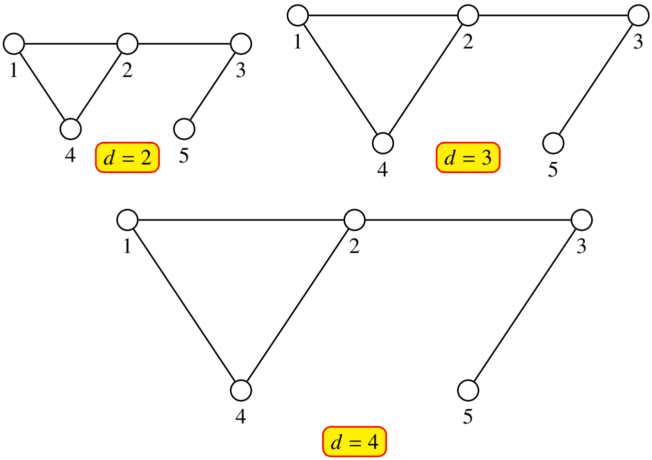
$d = 3$

$d = 4$

General Graphs

- Reduction from D2ISR under TJ on general graphs.

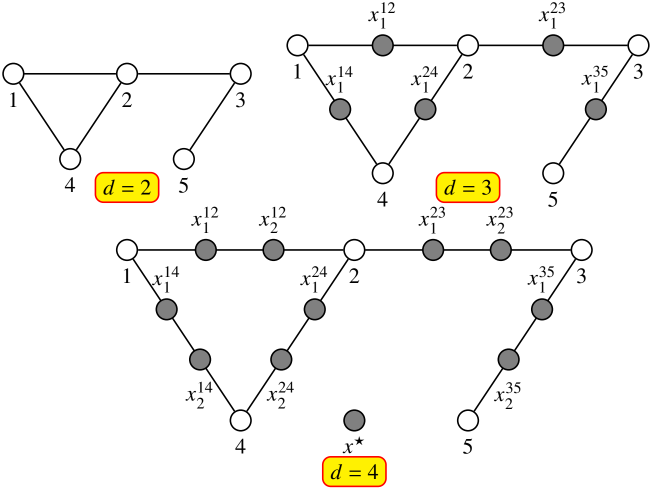
Figure: Reduction $(G, I, J, \mathsf{TJ}, 2) \Rightarrow (G', I, J, \mathsf{TJ}, d)$.



General Graphs

- Reduction from D2ISR under TJ on general graphs.

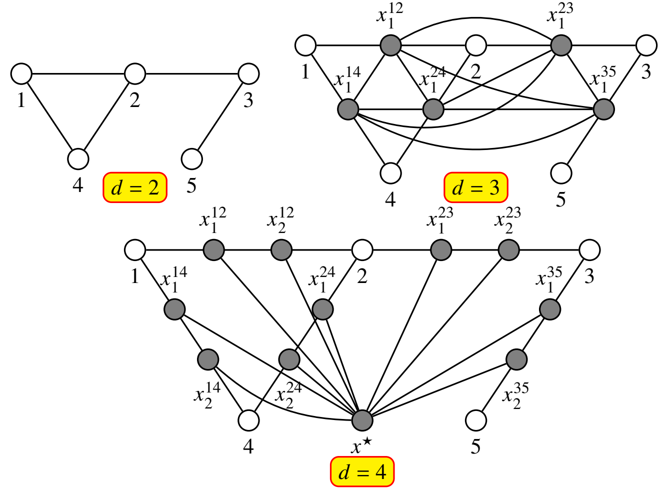
Figure: Reduction $(G, I, J, \mathsf{TJ}, 2) \Rightarrow (G', I, J, \mathsf{TJ}, d)$. Gray vertices are in $V(G') - V(G)$.



General Graphs

- Reduction from D2ISR under TJ on general graphs.

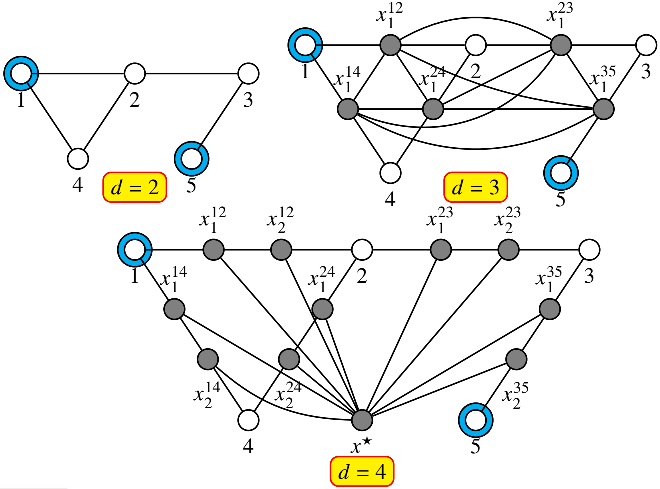
Figure: Reduction $(G, I, J, \mathsf{TJ}, 2) \Rightarrow (G', I, J, \mathsf{TJ}, d)$. Gray vertices are in $V(G') - V(G)$.



General Graphs

- Reduction from D2ISR under TJ on general graphs.

Figure: Reduction $(G, I, J, \mathsf{TJ}, 2) \Rightarrow (G', I, J, \mathsf{TJ}, d)$. Gray vertices are in $V(G') - V(G)$.



Claim 4

$(G, I, J, \mathsf{TJ}, 2)$ is YES $\Leftrightarrow$ $(G', I, J, \mathsf{TJ}, d)$ is YES.

Chordal Graphs and Split Graphs

Chordal Graphs and Split Graphs

Proposition 5

DdISR under TJ *on chordal graphs is in*

(a) P *for any even* $d \geq 2$

(b) PSPACE-*complete for any odd* $d \geq 3$

Chordal Graphs and Split Graphs

Proposition 5

DdISR under TJ *on chordal graphs is in*

(a) P *for any even* $d \geq 2$

(b) PSPACE*-complete for any odd* $d \geq 3$

(a) Reduce to *solving for* $d = 2$ *on chordal graphs*

- Any odd power of a chordal graph is also chordal [Balakrishnan and Paulraja 1983].
- DdISR under TJ on G is YES $\Leftrightarrow$ D2ISR under TJ on G^{d-1} is YES.
- D2ISR on chordal graphs is in P [Kamiński et al. 2012].

Chordal Graphs and Split Graphs

Proposition 5

DdISR under TJ *on chordal graphs is in*

(a) P *for any even $d \geq 2$*

(b) PSPACE*-complete for any odd $d \geq 3$*

(a) Reduce to *solving for $d = 2$ on chordal graphs*

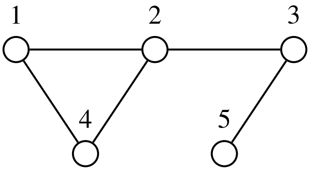
- Any odd power of a chordal graph is also chordal [Balakrishnan and Paulraja 1983].
- DdISR under TJ on G is YES $\Leftrightarrow$ D2ISR under TJ on G^{d-1} is YES.
- D2ISR on chordal graphs is in P [Kamiński et al. 2012].

(b) *Reduction from D2ISR under* TJ *on general graphs*—a PSPACE-complete whose complexity was shown in [Ito et al. 2011].

- Similar to *the reduction used by Eto et al.'s [Eto et al. 2014]* to show NP-completeness of MaxDdIS on chordal graphs.

Chordal Graphs and Split Graphs

Figure: Reduction $(G, I, J, \mathsf{TJ}, 2) \Rightarrow (G', I', J', \mathsf{TJ}, d)$ for odd $d \geq 3$.

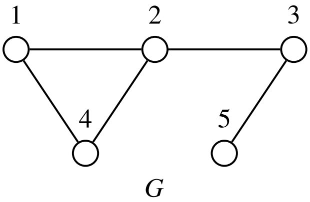


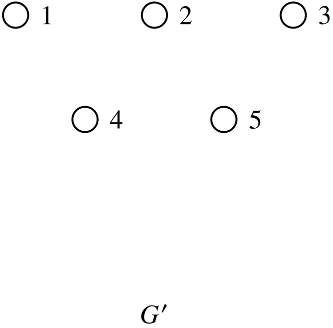
G

G'

Chordal Graphs and Split Graphs

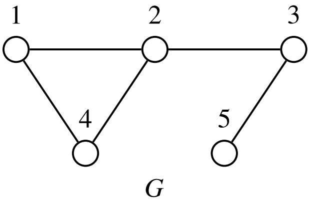
Figure: Reduction $(G, I, J, \mathsf{TJ}, 2) \Rightarrow (G', I', J', \mathsf{TJ}, d)$ for odd $d \geq 3$.

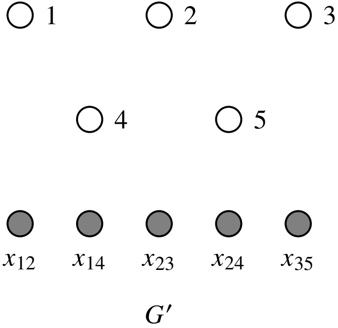




Chordal Graphs and Split Graphs

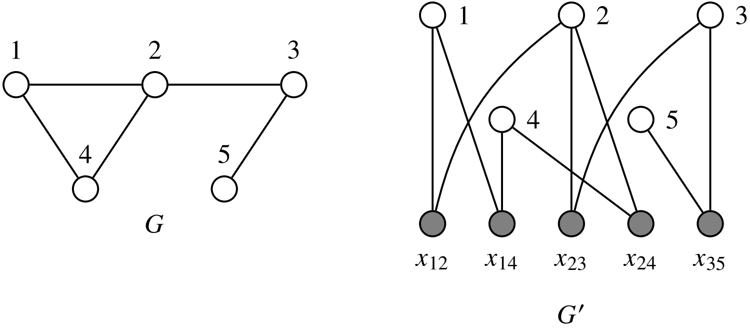
Figure: Reduction $(G, I, J, \mathsf{TJ}, 2) \Rightarrow (G', I', J', \mathsf{TJ}, d)$ for odd $d \geq 3$. Gray vertices are in $V(G') - V(G)$.





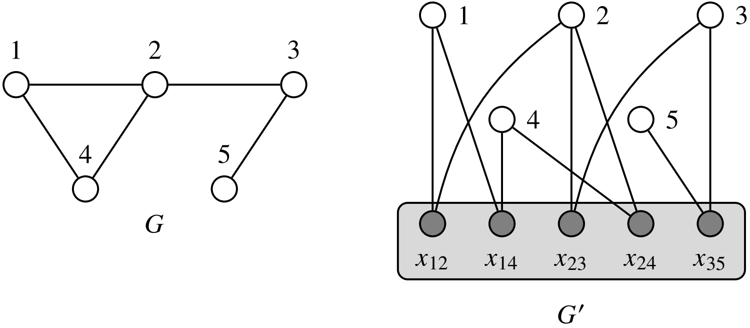
Chordal Graphs and Split Graphs

Figure: Reduction $(G, I, J, \mathsf{TJ}, 2) \Rightarrow (G', I', J', \mathsf{TJ}, d)$ for odd $d \geq 3$. Gray vertices are in $V(G') - V(G)$.



Chordal Graphs and Split Graphs

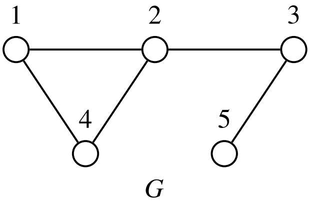
Figure: Reduction $(G, I, J, \mathsf{TJ}, 2) \Rightarrow (G', I', J', \mathsf{TJ}, d)$ for odd $d \geq 3$. Gray vertices are in $V(G') - V(G)$. Vertices in the light-gray box forms a clique.

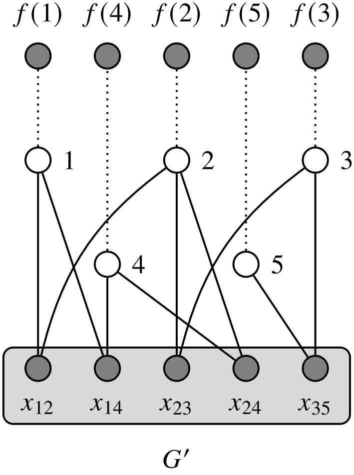


Chordal Graphs and Split Graphs

Figure: Reduction $(G, I, J, \mathsf{TJ}, 2) \Rightarrow (G', I', J', \mathsf{TJ}, d)$ for odd $d \geq 3$. Gray vertices are in $V(G') - V(G)$. Vertices in the light-gray box forms a clique. Each dotted path is of length $(d-3)/2$.

$d = 3$: G' is a split graph

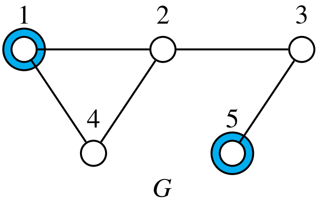


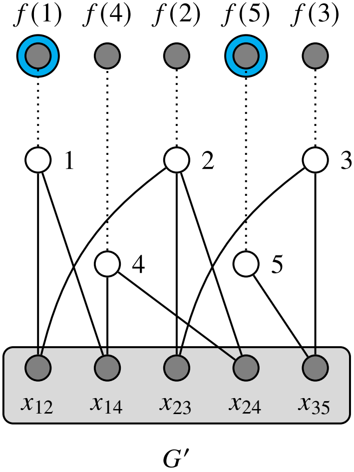


Chordal Graphs and Split Graphs

Figure: Reduction $(G, I, J, \mathsf{TJ}, 2) \Rightarrow (G', I', J', \mathsf{TJ}, d)$ for odd $d \geq 3$. Gray vertices are in $V(G') - V(G)$. Vertices in the light-gray box forms a clique. Each dotted path is of length $(d-3)/2$.

$d = 3$: G' is a split graph

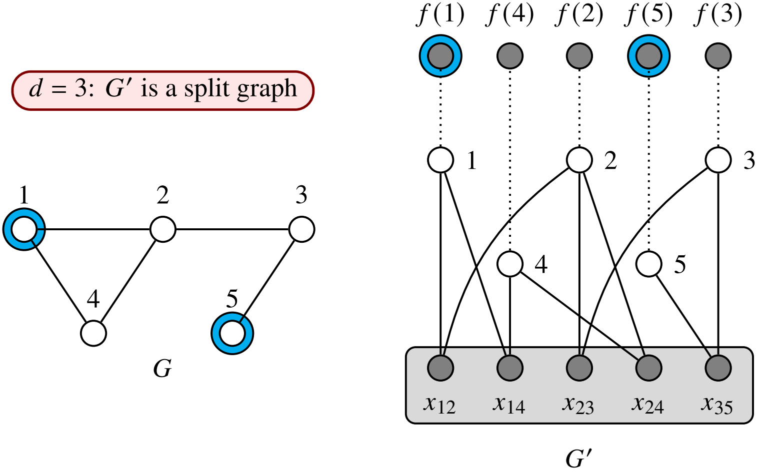




Chordal Graphs and Split Graphs

Figure: Reduction $(G, I, J, \mathsf{TJ}, 2) \Rightarrow (G', I', J', \mathsf{TJ}, d)$ for odd $d \geq 3$. Gray vertices are in $V(G') - V(G)$. Vertices in the light-gray box forms a clique. Each dotted path is of length $(d-3)/2$.

$d = 3$: G' is a split graph



Claim 6

$(G, I, J, \mathsf{TJ}, 2)$ is YES $\Leftrightarrow$ $(G', I', J', \mathsf{TJ}, d)$ is YES.

Chordal Graphs and Split Graphs

Proposition 7

DdISR under TJ *on split graphs is in*

(a) PSPACE*-complete for* $d = 3$

(b) P *for any* $d \neq 3$

(a) Consequence of the reduction on chordal graphs.

(b) The case $d = 2$ was proved in [Kamiński et al. 2012]. We observed for $d \geq 4$ before.

Chordal Graphs and Split Graphs

Proposition 7

DdISR under TJ *on split graphs is in*

(a) PSPACE*-complete for* $d = 3$

(b) P *for any* $d \neq 3$

(a) Consequence of the reduction on chordal graphs.

(b) The case $d = 2$ was proved in [Kamiński et al. 2012]. We observed for $d \geq 4$ before.

Proposition 8

DdISR under TS *on split graphs is in*

(a) PSPACE*-complete for* $d = 2$

(b) P *for any* $d \geq 3$

(a) [Belmonte et al. 2021].

(b) When $d = 3$ and each token-set has at least two members, no token can be moved. We observed for $d \geq 4$ before.

Open Question: Trees

Open Question: Trees

Proposition 9

DdISR under TJ *on trees is in* P.

- The power of a tree is a chordal graph [Lin and Skiena 1995].
- D2ISR under TJ on chordal graphs is in P [Kamiński et al. 2012].

Open Question: Trees

Proposition 9

DdISR under TJ *on trees is in* P.

- The power of a tree is a chordal graph [Lin and Skiena 1995].
- D2ISR under TJ on chordal graphs is in P [Kamiński et al. 2012].

Proposition 10: [Demaine et al. 2015]

D2ISR under TS *on trees is in* P.

- For a D2IS I of a tree T, Demaine et al. defined *(T, I)-rigid tokens*—the tokens that "cannot be moved at all".
- *Crucial points* leading to their algorithm:
  1. For any D2IS I, all (T, I)-rigid tokens can be found in polynomial time.
  2. For two D2ISs I and J, if (T, I)-rigid tokens and (T, J)-rigid tokens are the same, then I can be reconfigured to J under TS and vice versa.

Open Question: Trees

Proposition 9

DdISR under TJ *on trees is in* P.

- The power of a tree is a chordal graph [Lin and Skiena 1995].
- D2ISR under TJ on chordal graphs is in P [Kamiński et al. 2012].

Proposition 10: [Demaine et al. 2015]

D2ISR under TS *on trees is in* P.

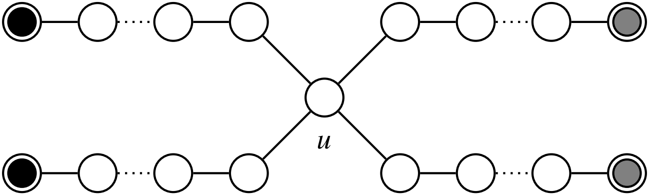
- For a D2IS I of a tree T, Demaine et al. defined *(T, I)-rigid tokens*—the tokens that "cannot be moved at all".
- *Crucial points* leading to their algorithm:
  1. For any D2IS I, all (T, I)-rigid tokens can be found in polynomial time.
  2. For two D2ISs I and J, if (T, I)-rigid tokens and (T, J)-rigid tokens are the same, then I can be reconfigured to J under TS and vice versa. [Not hold when $d \geq 3$]

Open Question: Trees

Observation

There are two DdISs I and J ($d \geq 3$) such that I cannot be reconfigured to J under TS even when the sets of (T, I)-rigid tokens and (T, J)-rigid tokens are both empty.

Figure: I cannot be reconfigured into J under TS ($d \geq 3$) even when there are no (T, I)-rigid and (T, J)-rigid tokens. Tokens in I (resp., J) are marked with the black (resp. gray) color. All tokens are of distance $d - 1$ from u.



Note

Demaine et al.'s approach *cannot* be applied.

References

Belmonte, R., E. J. Kim, M. Lampis, V. Mitsou, Y. Otachi, and F. Sikora (2021). "Token sliding on split graphs". In: *Theory of Computing Systems* 65.4, pp. 662–686. DOI: 10.1007/s00224-020-09967-8.

Katsikarelis, I., M. Lampis, and V. T. Paschos (2020). "Structurally parameterized d-scattered set". In: *Discrete Applied Mathematics* 308, pp. 168–186. DOI: 10.1016/j.dam.2020.03.052.

Lokshtanov, D. and A. E. Mouawad (2019). "The complexity of independent set reconfiguration on bipartite graphs". In: *ACM Transactions on Algorithms* 15.1, 7:1–7:19. DOI: 10.1145/3280825.

Yamanaka, K., S. Kawaragi, and T. Hirayama (2019). "Exact Exponential Algorithm for Distance-3 Independent Set Problem". In: *IEICE Transactions on Information and Systems* 102.3, pp. 499–501. DOI: 10.1587/transinf.2018FCL0002.

References

Jena, S. K., R. K. Jallu, G. K. Das, and S. C. Nandy (2018). "The maximum distance-d independent set problem on unit disk graphs". In: *Proceedings of FAW 2018*. Ed. by J. Chen and P. Lu. Vol. 10823. LNCS. Springer, pp. 68–80. DOI: 10.1007/978-3-319-78455-7_6.

Mouawad, A. E., N. Nishimura, V. Raman, and S. Siebertz (2018). "Vertex Cover Reconfiguration and Beyond". In: *Algorithms* 11.2. (article 20). DOI: 10.3390/a11020020.

Bonamy, M. and N. Bousquet (2017). "Token sliding on chordal graphs". In: *Proceedings of WG 2017*. Vol. 10520. LNCS. Springer, pp. 127–139. DOI: 10.1007/978-3-319-68705-6_10.

Bonsma, P. S. (2016). "Independent set reconfiguration in cographs and their generalizations". In: *Journal of Graph Theory* 83.2, pp. 164–195. DOI: 10.1002/jgt.21992.

References

- Hoang, D. A. and R. Uehara (2016). "Sliding Tokens on a Cactus". In: *Proceedings of ISAAC 2016*. Vol. 64. LIPIcs. Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 37:1–37:26. DOI: 10.4230/LIPIcs.ISAAC.2016.37.
- Montealegre, P. and I. Todinca (2016). "On distance-d independent set and other problems in graphs with "few" minimal separators". In: *Proceedings of WG 2016*. Ed. by P. Heggernes. Vol. 9941. LNCS. Springer, pp. 183–194. DOI: 10.1007/978-3-662-53536-3_16.
- Demaine, E. D., M. L. Demaine, E. Fox-Epstein, D. A. Hoang, T. Ito, H. Ono, Y. Otachi, R. Uehara, and T. Yamada (2015). "Linear-time algorithm for sliding tokens on trees". In: *Theoretical Computer Science* 600, pp. 132–142. DOI: 10.1016/j.tcs.2015.07.037.
- Fox-Epstein, E., D. A. Hoang, Y. Otachi, and R. Uehara (2015). "Sliding token on bipartite permutation graphs". In: *Proceedings of ISAAC 2015*. Vol. 9472. LNCS. Springer, pp. 237–247. DOI: 10.1007/978-3-662-48971-0_21.

References

- Bonsma, P. S., M. Kamiński, and M. Wrochna (2014). "Reconfiguring independent sets in claw-free graphs". In: *Proceedings of SWAT 2014*. Vol. 8503. LNCS. Springer, pp. 86–97. DOI: 10.1007/978-3-319-08404-6_8.
- Eto, H., F. Guo, and E. Miyano (2014). "Distance-d independent set problems for bipartite and chordal graphs". In: *Journal of Combinatorial Optimization* 27.1, pp. 88–99. DOI: 10.1007/s10878-012-9594-4.
- Kamiński, M., P. Medvedev, and M. Milanič (2012). "Complexity of independent set reconfigurability problems". In: *Theoretical Computer Science* 439, pp. 9–15. DOI: 10.1016/j.tcs.2012.03.004.
- Ito, T., E. D. Demaine, N. J. A. Harvey, C. H. Papadimitriou, M. Sideri, R. Uehara, and Y. Uno (2011). "On the complexity of reconfiguration problems". In: *Theoretical Computer Science* 412.12-14, pp. 1054–1065. DOI: 10.1016/j.tcs.2010.12.005.

References

Hearn, R. A. and E. D. Demaine (2005). "PSPACE-completeness of sliding-block puzzles and other problems through the nondeterministic constraint logic model of computation". In: *Theoretical Computer Science* 343.1-2, pp. 72–96. DOI: 10.1016/j.tcs.2005.05.008.

Lin, Y.-L. and S. S. Skiena (1995). "Algorithms for square roots of graphs". In: *SIAM Journal on Discrete Mathematics* 8.1, pp. 99–118. DOI: 10.1137/S089548019120016X.

Kong, M. and Y. Zhao (1993). "On computing maximum k-independent sets". In: *Congressus Numerantium* 95, pp. 47–47.

Balakrishnan, R. and P. Paulraja (1983). "Powers of chordal graphs". In: *Journal of the Australian Mathematical Society* 35.2, pp. 211–217. DOI: 10.1017/S1446788700025696.

Garey, M. R. and D. S. Johnson (1979). *Computers and intractability: A guide to the theory of NP-completeness*. W.H. Freeman and Company.

References

Gavril, F. (1972). “Algorithms for minimum coloring, maximum clique, minimum covering by cliques, and maximum independent set of a chordal graph”. In: *SIAM Journal on Computing* 1.2, pp. 180–187. DOI: 10.1137/0201013.